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ABSTRACT

Several empirical correlations between damping ratio with various factors are available and many researchers have developed empirical correlations between damping ratio and normalized shear modulus. Regardless of the appearance of the equations, the estimation of the damping ratio from the normalized shear modulus at any given shear strain amplitude is possible. In this paper, thirty-two sets of normalized shear modulus versus damping ratio from ten references are used to find a new empirical equation for damping ratio of sandy soils. For clayey soils, more than forty sets of normalized shear modulus versus damping ratio from nine references are used. After compiling and analyzing these previous studies and data, equations of sandy and clayey soils for the correlation are determined with coefficient of determination (r^2) = 0.918 and 0.844, respectively.

INTRODUCTION

Several empirical correlations between damping ratio (D) with various factors are available. Hardin (1965) proposed an empirical equation for damping ratio of clean dry sands for shear strain amplitudes (γ) in the order of 10^{-4}% to 10^{-2}%. For confining pressures ($\overline{\sigma}_0$) between 500 psf and 3,000 psf and frequencies less than 600 Hz, the damping ratio (D) were recommended to be calculated as:

$$D = 4.5\,\gamma^{0.2}\,\overline{\sigma}_0^{\,-0.5} \tag{1}$$

Sherif et al. (1977) established another empirical equation for damping ratio based on cyclic torsional shear tests on dry Ottawa sand at second loading cycle as:

$$D\,(\%) = \left(50 - 60\,\overline{\sigma}_c\right)\cdot \gamma\,(\%)^{0.3} \tag{2}$$

where $\overline{\sigma}_c$ is effective confining pressure in psi. They modified Equation 2 further to include the effects of soil gradation and number of cycles as:

$$\begin{aligned} D\,(\%) = &\left(\left(50 - 60\,\overline{\sigma}_c\right)/38\right)\cdot(73F - 53) \\ &\times (1.01 - 0.046\log N)\cdot \gamma\,(\%)^{0.3} \end{aligned} \tag{3}$$

where N is number of loading cycles, and F is soil gradation and sphericity factor, which is functions of soil sphericity and coefficient of curvature. Typically, the value of F varies from 1.0 to 2.0 (Saxena and Reddy, 1989).

Edil and Luh (1978) observed a significant effect of the number of cycles on damping ratio for Ottawa sand and other dry sands, and developed the correlation as:

$$D/D_0 = 1.131 - 0.0453 \log N \quad (4)$$

and

$$D_0 = 0.88 + 6592\gamma \cdot e^{0.5} - 0.28\gamma^{0.33} \times (\bar{\sigma}_0/98.07)^3 - 73.55\gamma \cdot \bar{\sigma}_0{}^{0.5} \quad (5)$$

where D_0 is the damping ratio determined at number of cycles (N) equal to 1000, D_0 and D are in percentage, e is void ratio, and $\bar{\sigma}_0$ is expressed in kPa. Expressions for D_0 in terms of relative density instead of using void ratio were also developed.

Adopting the observations of Hardin (1965) and Sherif et al. (1977), Saxena and Reddy (1989) expressed the damping ratio amplification of Monterey No.0 sand as:

$$D(\%) = 9.22\gamma^{0.33} (\bar{\sigma}_0/P_a)^{-0.38} \quad (6)$$

where P_a is the atmospheric pressure. The unit of $\bar{\sigma}_0$ and P_a should be the same, and shear strain amplitude (γ) is expressed in percentage.

METHODOLOGY

As can be observed above, shear strain is the key parameter for most empirical equations. After compiling available data for saturated clays from twelve references, Seed and Idriss (1970) suggested typical shear reduction curve and damping ratio with shear strain. These relationships may be displayed more clearly in 3-D diagram as shown in Fig. 1. It shows that both properties (G/G_{max} and D) depend on shear strain, and it may be possible to demonstrate a relationship between these two properties. Indeed, empirical equations for damping ratio using G/G_{max} seem to be most convenient and widely used, since both modulus ratio and damping ratio depend, in general, on the same parameters. For example, it was found that plasticity index (PI) has significant effects on both normalized shear modulus and damping ratio. It was demonstrated that it was not an easy task to formulate the

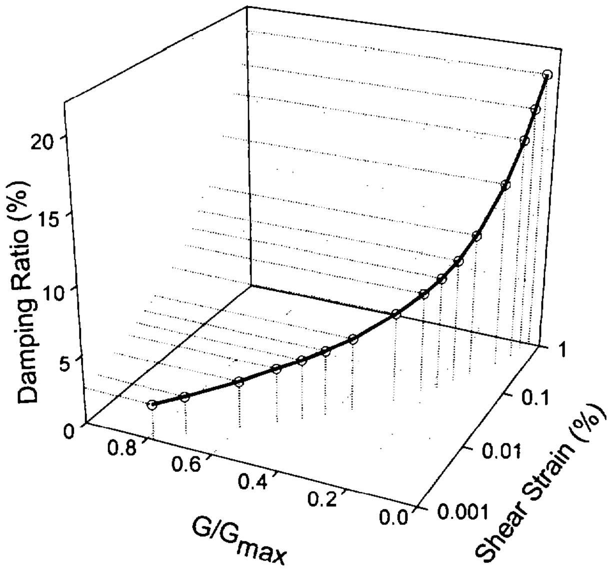

Fig. 1. Relationship between normalized shear modulus and damping ratio based on shear strain for saturated clays (data from Seed and Idriss, 1970).

effect of PI on G/G_{max} and D separately. However, PI effect does not have to be included in the formulation for damping ratio when the equation is based on the normalized shear modulus. This statement is justifiable, since the trend and shape of damping change with PI are systematically similar to those of modulus reduction change with PI. Figure 2 illustrates the relationship between damping ratio (D) and modulus ratio (G/G_{max}), which was a function of inversely proportional at various PI.

Many researchers have developed empirical correlations between damping ratio and normalized shear modulus. Hardin and Drnevich (1972) derived a simple relationship between modulus and damping ratio as:

$$D = D_{max} (1 - (G/G_{max})) \quad (7)$$

Tatsuoka et al. (1978) also investigated the relationship between shear modulus and damping ratio. They found decreasing trend of damping ratio with increasing shear modulus ratio (G/G_{max}) qualitatively. Damping ratio could be expressed as a function of the normalized shear modulus ratio (G/G_{max}) as:

$$D = f(G/G_{max}) \quad (8)$$

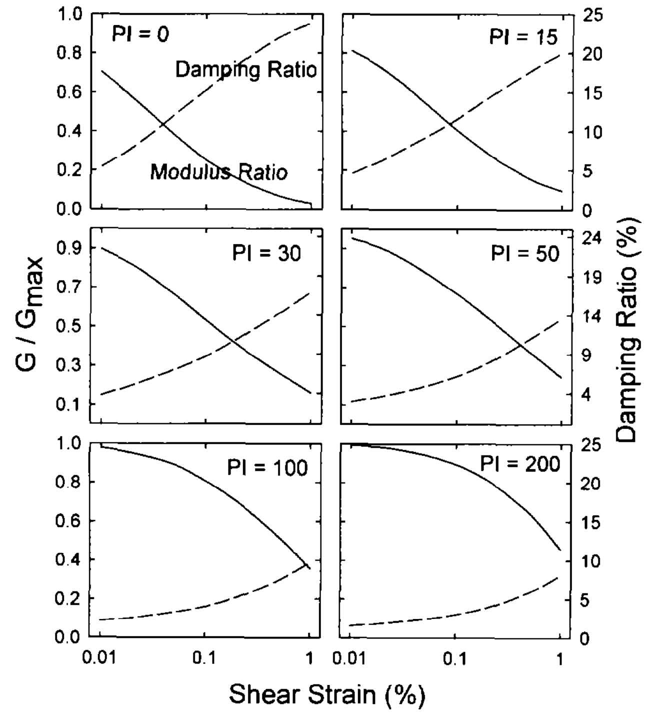

Fig. 2. Systematic change of damping ratio (D) with normalized shear modulus (G/G_{max}) for various plasticity indices (modified from Vucetic and Dobry, 1991).

Using the idea of Equation 8, several different forms of equations were proposed. Uchida et al. (1980) proposed the relationship between damping ratio and shear modulus ratio as:

$$D = (1-(G/G_{max}))\cdot(a + b(G/G_{max})) \quad (9)$$

where the values of a and b are determined from curve fitting for a given soil. By analyzing test data from previous research on sands, Khouri (1984), and Ishibashi and Zhang (1993) proposed an equation of the damping ratio (D) as:

$$D = 0.333\left[0.586\left(\frac{G}{G_{max}}\right)^2 - 1.547\left(\frac{G}{G_{max}}\right) + 1\right] \quad (10)$$

In Equation 10, a maximum damping ratio (D_{max}) was defined as D_{max} = 33.3%, which corresponds to very high shear strain levels where the G/G_{max} ratio is nearly equal to zero. D_{max} = 33.3% is a representative value from many previous researchers (Hardin and Drnevich, 1972; Sherif et al., 1977; Tatsuoka et al., 1978) for sands. Zhang (1994) proposed another equation as:

$$D(\%) = A\cdot \exp^{-a\,(G/G_{max})^b} \quad (11)$$

where A, a, and b are the regression parameters.

Borden et al. (1996) proposed another equation as:

$$D(\%) = 20.4\left((G/G_{max})-1\right)^2 + 3.1 \quad (12)$$

Regardless of the appearance of the equations, the estimation of the damping ratio from the normalized shear modulus at any given shear strain amplitude is possible.

ANALYSES

In this study, thirty-two sets of normalized shear modulus versus damping ratio from ten references are used to find a new empirical equation for damping ratio of sandy soils. For clayey soils, forty-eight sets of normalized shear modulus versus and damping ratio from nine references are used. Table 1 and 2 summarize the references used for sandy and clayey soils, respectively.

Figure 3 shows the damping ratio (D) change with normalized shear modulus. It shows a clear relationship between D and G/G_{max} for sandy soils. An equation for the correlation is determined as:

$$D(\%) = 32.85\left[0.54\left(\frac{G}{G_{max}}\right)^2 - 1.53\left(\frac{G}{G_{max}}\right) + 1\right] \quad (13)$$

with coefficient of determination (r^2) = 0.918. Several other functions such as linear, sigmoidal, modified hyperbolic, power, and rational were tried to find best fitting curve. Most of the functions yield the r^2 more than 0.90 and quadratic polynomial function results in the best correlation. As can be noticed in Equation 13, D_{max} would be about 32.8% when G/G_{max} is zero and D_{min} is calculated as about 0.3% when G/G_{max} is one.

Similarly, the damping ratio of clayey soils versus normalized shear modulus is plotted in Fig. 4, and can be represented by

Table 1 Summary of references used for sandy soils.

Reference	Sand Type	Index
Kokusho (1980)	Toyoura sand	a
Kokusho and Esashi (1981)	Toyoura sand	b
Alarcon-Guzman (1986)	Ottawa sand	c
Ni (1987)	washed mortar sand	d
Saxena and Reddy (1989)	Monterey No.0 sand	e, f *
Yasuda and Matsumoto (1993)	Toyoura sand	g
Hardin et al. (1994)	Ottawa sand	h
Kanatani et al. (1994)	Toyoura sand	i
Macari and Ko (1994)	non-plastic marine silt	j
Yamashita and Toki (1994)	Ishikari sand	k

* e for $d_r = 25\%$ and f for $d_r = 60\%$

Table 2 Summary of references used for clayey soils.

Reference	Clay Type	Index
Stokoe et al. (1980)	offshore clayey silt	a
Kokusho et al. (1982)	Teganuma and Hommoku clay	b
Puri (1984)	loessial silty clay	c, d, e *
Saada (1985)	various clays	f, g *
Tawfiq (1986)	Edger plastic kaolin	h
Diaz-Rodriguez (1992)	Mexico City clay	i
Pitilakis et al. (1992)	Greek silty clay	j
Zavoral and Campanella (1994)	UBC clay	k
Guha (1995)	San Francisco Old Bay clay	l

* c for reconstituted, d for S = 100%, e for S = 74-78%, f for various anisotropic, and g for isotropic

an empirical equation as:

$$D(\%) = 17.83\left[0.56\left(\frac{G}{G_{max}}\right)^2 - 1.39\left(\frac{G}{G_{max}}\right) + 1\right] \qquad (14)$$

with coefficient of determination (r^2) = 0.844.

Several other functions were also tried to find best fitting curve. Most of the functions yield r^2 less than 0.84, which indicates more scattered data compared to sandy soils. The quadratic polynomial function provides reasonable correlation.

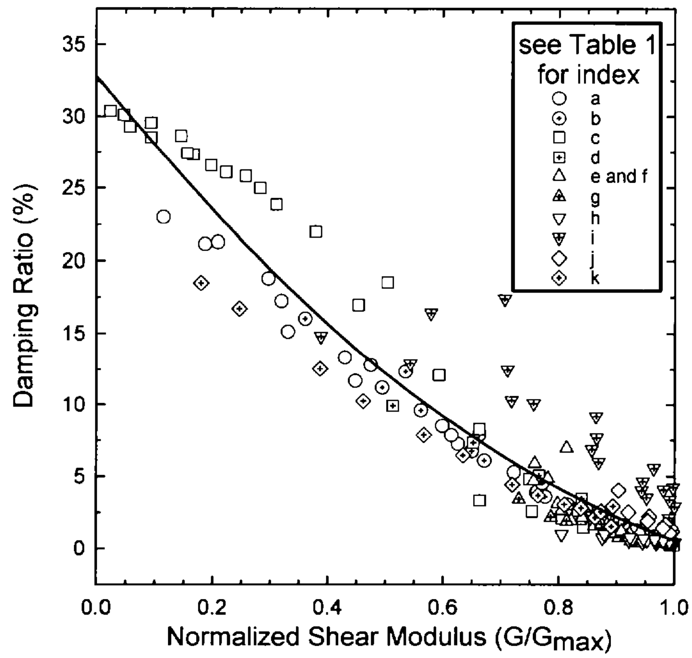

Fig. 3. Damping ratio versus normalized shear modulus for sandy soils.

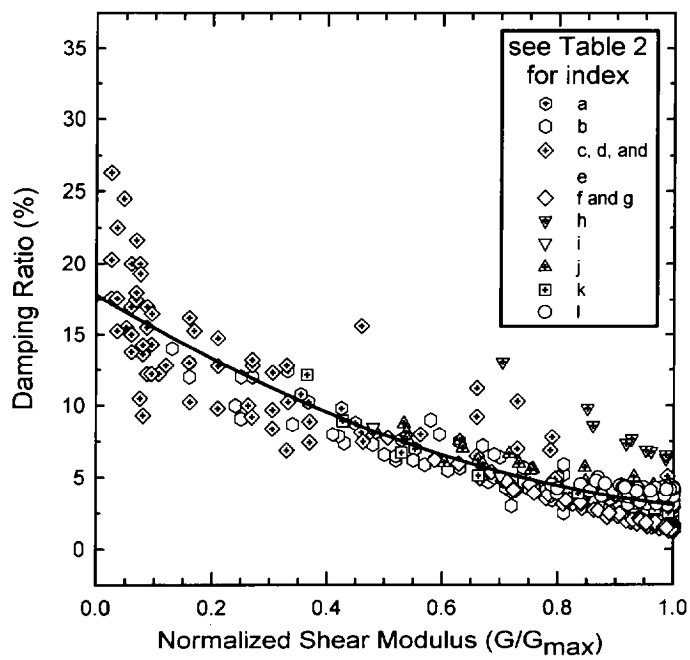

Fig. 4. Damping ratio versus normalized shear modulus for clayey soils.

As can be observed in Equation 14, D_{max} would be about 17.8% when G/G_{max} is zero and D_{min} is about 3.1% when G/G_{max} is one. The damping ratio of clayey soils is found to be smaller than that of sandy soils in most cases. However, D_{min} and the damping amplification until G/G_{max} reaches about 0.75 are found to be slightly larger than those of sandy soils from the result of regression analyses.

CONCLUSIONS

Regardless of the appearance of the equations proposed by previous researches, the estimation of the damping ratio from the normalized shear modulus at any given shear strain amplitude was possible. Since both modulus reduction curve and damping ratio depend on the same parameters generally and the damping amplification is inversely proportional to the modulus reduction curve, two empirical equations for damping ratio using G/G_{max} were proposed for sandy and clayey soils based on the previous studies.

It can be concluded that these equations can be used to determine approximate damping ratio when the normalized shear modulus at any shear strain is known.

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